Numbers and the number system — 12 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Number and Place Value progression map

- use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple
- use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions

Return to overview

Possible themes

- Solve problems using common factors and highest common factors
- Exploring prime numbers
- Solve problems using common multiples and lowest common multiples
- Explore powers and roots

Possible key learning points

- Find prime numbers and test numbers to see if they are prime
- Find common factors of numbers
- Find the highest common factor of numbers in simple cases, including co-prime examples
- Find common multiples of numbers
- Recognise and solve problems involving the lowest common multiple
- Use linear (arithmetic) number patterns to solve problems
- Recognise and use triangular numbers
- Recognise and use square and cube numbers
- Read, write and evaluate powers
- Define and find square roots (including using the $\sqrt{\ }$ symbol)
- Define and find cube roots (including using the $\sqrt[3]{\ }$ symbol), including the use of a scientific calculator
- Define and find other roots (including using the $\sqrt{\ }$ symbol), including the use of a scientific calculator

Prerequisites

- Know how to find common multiples of two given numbers
- Know how to find common factors of two given numbers
- Recall multiplication facts to 12 × 12 and associated division facts

Bring on the Maths⁺: Moving on up!
Number and Place Value: #6

Mathematical language

((Lowest) common) multiple and LCM
((Highest) common) factor and HCF
Power
(Square and cube) root
Triangular number, Square number, Cube number, Prime number
Linear sequence, Arithmetic sequence

Notation
Index notation: e.g. 5^3 is read as '5 to the power of 3' and means '3 lots of 5 multiplied together'
Radical notation: e.g. $\sqrt{49}$ is generally read as 'the square root of 49' and means 'the positive square root of 49'; $\sqrt[3]{8}$ means 'the cube root of 8'

Pedagogical notes

Pupils need to know how to use a scientific calculator to work out powers and roots.
Note that while the square root symbol (√) refers to the positive square root of a number, every positive number has a negative square root too.
NCETM: Departmental workshop: Index Numbers
NCETM: Glossary

Common approaches
The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
Every classroom has a set of number classification posters on the wall

Reasoning opportunities and probing questions

- When using Eratosthenes sieve to identify prime numbers, why is there no need to go further than the multiples of 7? If this method was extended to test prime numbers up to 200, how far would you need to go? Convince me.
- Kenny says '20 is a square number because $10^2 = 20$'. Explain why Kenny is wrong. Kenny is partially correct. How could he change his statement so that it is fully correct?
- Always / Sometimes / Never: The lowest common multiple of two numbers is found by multiplying the two numbers together.

Suggested activities

KM: Perfect numbers: includes use of factors, primes and powers
KM: Exploring primes activities: Factors of square numbers; Mersenne primes; LCM sequence; n^2 and $(n + 1)^2$; n^2 and $n^2 + n$; $n^2 + 1$; $n! + 1$; $n! – 1$; $x^2 + x + 41$
KM: Use the method of Eratosthenes' sieve to identify prime numbers, but on a grid 6 across by 17 down instead. What do you notice?
KM: Square number puzzle
KM: History and Culture: Goldbach's Conjectures
NRICH: Factors and multiples
NRICH: Powers and roots

Learning review
KM: 7M1 BAM Task

Possible misconceptions

- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'
- A common misconception is to believe that $5^3 = 5 \times 3 = 15$
- See pedagogical note about the square root symbol too

Calculating — 16 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Calculation progression map

- understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- apply the four operations, including formal written methods, to integers and decimals
- use conventional notation for priority of operations, including brackets
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

Return to overview

Possible themes

- Exploring place value
- Exploring written methods of calculation
- Calculating with decimals
- Know and apply the correct order of operations

Possible key learning points

- Multiply a positive integer by a power of 10
- Multiply a decimal by a power of 10
- Divide a positive integer by a power of 10
- Divide a decimal by a power of 10
- Add numbers up to six-digits using a formal written method
- Add decimals with the same, and different, number of decimal places
- Subtract numbers up to six-digits using a formal written method
- Subtract decimals with the same, and different, number of decimal places
- Multiply a number up to four-digits by a one or two-digit number using a formal written method
- Transform a multiplication involving decimals to a corresponding multiplication with integers
- Multiply a large integer up to four-digits by a decimal of up to 2dp using integer multiplication
- Divide a number up to four-digits by a one or two-digit number using a formal written method
- Use a formal method to divide a decimal by an integer < 10
- Use a formal method to divide a decimal by an integer greater than 10
- Transform a calculation involving the division of decimals to an equivalent division involving integers
- Apply the order of operations to multi-step calculations involving up to four operations and brackets

Prerequisites

- Fluently recall multiplication facts up to 12 × 12
- Fluently apply multiplication facts when carrying out division
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Know the formal written method of long division
- Convert between an improper fraction and a mixed number

Bring on the Maths+: Moving on up!
Calculating: #2, #3, #4, #5
Fractions, decimals & percentages: #6, #7
Solving problems: #2

Mathematical language

Improper fraction
Top-heavy fraction
Mixed number
Operation
Inverse
Long multiplication
Short division
Long division
Remainder

Pedagogical notes

Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g. 6 – 2 + 3.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
KM: Progression: Addition and Subtraction, Progression: Multiplication and Division and Calculation overview
NCETM: Departmental workshop: Place Value
NCETM: Subtraction, Multiplication, Division, Glossary

Common approaches
All classrooms display a times table poster with a twist
Long multiplication is promoted as the 'most efficient method'.
Short division is promoted as the 'most efficient method'.
If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.

Reasoning opportunities and probing questions

- Jenny says that 2 + 3 × 5 = 25. Kenny says that 2 + 3 × 5 = 17. Who is correct? How do you know?
- Find missing digits in otherwise completed long multiplication / short division calculations
- Show me a calculation that is connected to 14 × 26 = 364. And another. And another …

Suggested activities

KM: Long multiplication template
KM: Dividing (lots)
KM: Interactive long division
KM: Misplaced points
KM: 4 to 1 challenge
KM: Maths to Infinity: Multiplying and dividing
NRICH: Cinema Problem
NRICH: Funny factorisation
NRICH: Skeleton
NRICH: Long multiplication

Learning review
KM: 7M2 BAM Task

Possible misconceptions

- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.
- Pupils may incorrectly apply place value when dividing by a decimal for example by making the answer 10 times bigger when it should be 10 times smaller.
- Some pupils may have inefficient methods for multiplying and dividing numbers.

Checking, approximating and estimating — *3 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Number and Place Value progression map

- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

Return to overview

Possible themes

- Explore ways of approximating numbers
- Explore ways of checking answers

Possible key learning points

- Round a number to a specified number of decimal places
- Round a number to one significant figure
- Estimate calculations by rounding numbers to one significant figure

Prerequisites

- Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000, 100 000 or 1 000 000
- Approximate any number with one or two decimal places by rounding to the nearest whole number
- Approximate any number with two decimal places by rounding to the one decimal place
- Simplify a fraction by cancelling common factors

Mathematical language

Approximate (noun and verb)
Round
Decimal place
Check
Solution
Answer
Estimate (noun and verb)
Order of magnitude
Accurate, Accuracy
Significant figure
Cancel
Inverse
Operation

Notation

The approximately equal symbol (≈)
Significant figure is abbreviated to 's.f.' or 'sig fig'

Pedagogical notes

Pupils should be able to estimate calculations involving integers and decimals.
Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map
NCETM: Glossary

Common approaches

All pupils are taught to visualise rounding through the use a number line

Reasoning opportunities and probing questions

- Convince me that 39 652 rounds to 40 000 to one significant figure
- Convince me that 0.6427 does <u>not</u> round to 1 to one significant figure
- What is wrong: $\frac{11 \times 28.2}{0.54} \approx \frac{10 \times 30}{0.5} = 150$. How can you correct it?

Suggested activities

KM: Approximating calculations
KM: Stick on the Maths: CALC6: Checking solutions

Learning review

KM: 7M6 BAM Task

Possible misconceptions

- Some pupils may truncate instead of round
- Some pupils may round down at the half way point, rather than round up.
- Some pupils may think that a number between 0 and 1 rounds to 0 or 1 to one significant figure
- Some pupils may divide by 2 when the denominator of an estimated calculation is 0.5

Counting and comparing — 9 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Number and Place Value progression map

- order positive and negative integers, decimals and fractions
- use the symbols =, ≠, <, >, ≤, ≥

Return to overview

Possible themes

- Comparing numbers
- Ordering integers and decimals
- Ordering fractions
- Ordering integers, decimals and fractions (including mixed numbers)
- Using comparison symbols in algebraic contexts

Possible key learning points

- Use the signs <, > and = to compare numbers
- Use a compound inequality to compare three or more numbers (e.g. -1<0.5<4)
- Order a set of integers
- Order a set of decimals
- Order a set of integers and decimals
- Order fractions with the same denominator or denominators are a multiple of each other
- Order fractions where the denominators are not multiples of each other
- Order mixed numbers and fractions
- Order a combination of integers, decimals, fractions and mixed numbers

Prerequisites

- Understand that negative numbers are numbers less than zero
- Order a set of decimals with a mixed number of decimal places (up to a maximum of three)
- Order fractions where the denominators are multiples of each other
- Order fractions where the numerator is greater than 1
- Know how to simplify a fraction by cancelling common factors

Mathematical language

Positive number
Negative number
Integer
Numerator
Denominator

Notation

The 'equals' sign: =
The 'not equal' sign: ≠
The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)

Pedagogical notes

Zero is neither positive nor negative. The set of integers includes the natural numbers {1, 2, 3, …}, zero (0) and the 'opposite' of the natural numbers {-1, -2, -3, …}.
Pupil must use language correctly to avoid reinforcing misconceptions: for example, 0.45 should never be read as 'zero point forty-five'; 5 > 3 should be read as 'five is greater than 3', not '5 is bigger than 3'.
Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first.
The equals sign was designed by Robert Recorde in 1557 who also introduced the plus (+) and minus (-) symbols.
NCETM: Glossary

Common approaches

Teachers use the language 'negative number' to avoid future confusion with calculation that can result by using 'minus number'
Every classroom has a negative number washing line on the wall

Reasoning opportunities and probing questions

- Jenny writes down 0.400 > 0.58. Kenny writes down 0.400 < 0.58. Who do you agree with? Explain your answer.
- Find a fraction which is greater than 3/5 and less than 7/8. And another. And another …
- Convince me that -15 < -3

Suggested activities

KM: Inequality
KM: Farey Sequences
KM: Decimal ordering cards 2
KM: Maths to Infinity: Fractions, decimals and percentages
KM: Maths to Infinity: Directed numbers
NRICH: Greater than or less than?
YouTube: The Story of Zero

Possible misconceptions

- Some pupils may believe that 0.400 is greater than 0.58
- Pupils may believe, incorrectly, that:
  - A fraction with a larger denominator is a larger fraction
  - A fraction with a larger numerator is a larger fraction
  - A fraction involving larger numbers is a larger fraction
- Some pupils may believe that -6 is greater than -3. For this reason ensure pupils avoid saying 'bigger than'

Visualising and constructing — 4 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries
- use the standard conventions for labelling and referring to the sides and angles of triangles
- draw diagrams from written description

Return to overview

Possible themes

- Interpret geometrical conventions and notation
- Apply geometrical conventions and notation

Bring on the Maths+: Moving on up!
Properties of shapes: #3, #4

Possible key learning points

- Identify line and rotational symmetry in polygons
- Understand and use labelling notation for lengths and angles
- Use ruler and protractor to construct triangles, and other shapes, from written descriptions
- Use ruler and compasses to construct triangles when all three sides known

Prerequisites

- Use a ruler to measure and draw lengths to the nearest millimetre
- Use a protractor to measure and draw angles to the nearest degree

Mathematical language

Edge, Face, Vertex (Vertices)
Plane
Parallel
Perpendicular
Regular polygon
Rotational symmetry

Notation
The line between two points A and B is AB
The angle made by points A, B and C is $\angle ABC$
The angle at the point A is $\hat{A}$
Arrow notation for sets of parallel lines
Dash notation for sides of equal length

Pedagogical notes

NCETM: Departmental workshop: Constructions
The equals sign was designed (by Robert Recorde in 1557) based on two equal length lines that are equidistant
NCETM: Glossary

Common approaches
Dynamic geometry software to be used by all students to construct and explore dynamic diagrams of perpendicular and parallel lines.

Reasoning opportunities and probing questions

- Given SSS, how many different triangles can be constructed? Why? Repeat for ASA, SAS, SSA, AAS, AAA.
- Always / Sometimes / Never: to draw a triangle you need to know the size of three angles; to draw a triangle you need to know the size of three sides.
- Convince me that a hexagon can have rotational symmetry with order 2.

Suggested activities

KM: Shape work (selected activities)
KM: Rotational symmetry
NRICH: Notes on a triangle

Learning review
KM: 7M13 BAM Task

Possible misconceptions

- Two line segments that do not touch are perpendicular if they would meet at right angles when extended
- Pupils may believe, incorrectly, that:
  - perpendicular lines have to be horizontal / vertical
  - only straight lines can be parallel
  - all triangles have rotational symmetry of order 3
  - all polygons are regular

Investigating properties of shapes — *5 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

Return to overview

Possible themes

- Investigate the properties of 3D shapes
- Explore quadrilaterals
- Explore triangles

Possible key learning points

- Know the connection between faces, edges and vertices in 3D shapes
- Recognise and use nets of 3D shapes
- Know and solve problems using the properties and definitions of triangles
- Know and solve problems using the properties and definitions of special types of quadrilaterals (including diagonals)
- Know and solve problems using the properties of other plane figures

Prerequisites

- Know the names of common 3D shapes
- Know the meaning of face, edge, vertex
- Understand the principle of a net
- Know the names of special triangles
- Know the names of special quadrilaterals
- Know the meaning of parallel, perpendicular
- Know the notation for equal sides, parallel sides, right angles

Bring on the Maths⁺: Moving on up!
Properties of shapes: #1, #2

Mathematical language

Face, Edge, Vertex (Vertices)
Cube, Cuboid, Prism, Cylinder, Pyramid, Cone, Sphere
Quadrilateral
Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus
Delta, Arrowhead
Diagonal
Perpendicular
Parallel
Triangle
Scalene, Right-angled, Isosceles, Equilateral

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Right angle notation

Pedagogical notes

Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape.
A cube is a special case of a cuboid and a rhombus is a special case of a parallelogram
A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.
NCETM: Departmental workshop: 2D shapes
NCETM: Glossary

Common approaches
Every classroom has a set of triangle posters and quadrilateral posters on the wall
Models of 3D shapes to be used by all students during this unit of work

Reasoning opportunities and probing questions

- Show me an example of a trapezium. And another. And another …
- Always / Sometimes / Never: The number of vertices in a 3D shape is greater than the number of edges
- Which quadrilaterals are special examples of other quadrilaterals? Why? Can you create a 'quadrilateral family tree'?
- What is the same and what is different: Rhombus / Parallelogram?

Suggested activities

KM: Euler's formula
KM: Visualising 3D shapes
KM: Complete the net
KM: Dotty activities: Shapes on dotty paper
KM: What's special about quadrilaterals? Constructing quadrilaterals from diagonals and summarising results.
NRICH: A chain of polyhedra
NRICH: Property chart
NRICH: Quadrilaterals game

Possible misconceptions

- Some pupils may think that all trapezia are isosceles
- Some pupils may think that a diagonal cannot be horizontal or vertical
- Two line segments that do not touch are perpendicular if they would meet at right angles when extended. Therefore the diagonals of an arrowhead (delta) are perpendicular despite what some pupils may think
- Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond
- The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think

Algebraic proficiency: tinkering

8 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- understand and use the concepts and vocabulary of expressions, equations, formulae and terms
- use and interpret algebraic notation, including: ab in place of a × b, 3y in place of y + y + y and 3 × y, a^2 in place of a × a, a^3 in place of a × a × a, a/b in place of a ÷ b, brackets
- simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket
- where appropriate, interpret simple expressions as functions with inputs and outputs
- substitute numerical values into formulae and expressions
- use conventional notation for priority of operations, including brackets

Return to overview

Possible themes

- Understand the vocabulary and notation of algebra
- Manipulate algebraic expressions
- Explore functions
- Evaluate algebraic statements

Possible key learning points

- Know the meaning of expression, term, formula, equation, function
- Know and use basic algebraic notation (the 'rules' of algebra)
- Simplify a simple expression by collecting like terms
- Simplify more complex expressions by collecting like terms
- Manipulate expressions by multiplying an integer over a bracket (the distributive law)
- Manipulate expressions by multiplying a single term over a bracket (the distributive law)
- Substitute positive numbers into expressions and formulae
- Given a function, establish outputs from given inputs and inputs from given outputs

Prerequisites

- Use symbols (including letters) to represent missing numbers
- Substitute numbers into worded formulae
- Substitute numbers into simple algebraic formulae
- Know the order of operations

Bring on the Maths⁺: Moving on up!
Algebra: #1

Mathematical language

Algebra
Expression, Term, Formula (formulae), Equation, Function, Variable
Mapping diagram, Input, Output
Represent
Substitute
Evaluate
Like terms
Simplify / Collect

Notation
See Key concepts (GCSE subject content statements) above

Pedagogical notes

Pupils will have experienced some algebraic ideas previously. Ensure that there is clarity about the distinction between representing a variable and representing an unknown.
Note that each of the statements 4x, 42 and 4½ involves a different operation after the 4, and this can cause problems for some pupils when working with algebra.
NCETM: Algebra
NCETM: Glossary

Common approaches
All pupils are expected to learn about the connection between mapping diagrams and formulae (to represent functions) in preparation for future representations of functions graphically.

Reasoning opportunities and probing questions

- Show me an example of an expression / formula / equation
- Always / Sometimes / Never: 4(g+2) = 4g+8, 3(d+1) = 3d+1, a^2 = 2a, ab = ba
- What is wrong?
- Jenny writes 2a + 3b + 5a – b = 7a + 3. Kenny writes 2a + 3b + 5a – b = 9ab. What would you write? Why?

Suggested activities

KM: Pairs in squares. Prove the results algebraically.
KM: Algebra rules
KM: Use number patterns to develop the multiplying out of brackets
KM: Algebra ordering cards
KM: Spiders and snakes. See the 'clouding the picture' approach
KM: Maths to Infinity: Brackets
NRICH: Your number is …
NRICH: Crossed ends
NRICH: Number pyramids and More number pyramids

Learning review
KM: 7M7 BAM Task, 7M8 BAM Task, 7M9 BAM Task

Possible misconceptions

- Some pupils may think that it is always true that a=1, b=2, c=3, etc.
- A common misconception is to believe that a^2 = a × 2 = a2 or 2a (which it can do on rare occasions but is not the case in general)
- When working with an expression such as 5a, some pupils may think that if a=2, then 5a = 52.
- Some pupils may think that 3(g+4) = 3g+4
- The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x' may cause some confusion. In particular some pupils may think that 5h – h = 5

Exploring fractions, decimals and percentages — 4 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Fractions, decimals and percentages progression map

- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- define percentage as 'number of parts per hundred'
- express one quantity as a percentage of another

Return to overview

Possible themes

- Understand and use top-heavy fractions
- Understand the meaning of 'percentage'
- Explore links between fractions and percentages

Possible key learning points

- Write one quantity as a fraction of another where the fraction is less than 1
- Write one quantity as a fraction of another where the fraction is greater than 1
- Write a percentage as a fraction
- Write a quantity as a percentage of another

Prerequisites

- Understand the concept of a fraction as a proportion
- Understand the concept of equivalent fractions
- Understand the concept of equivalence between fractions and percentages

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #1, #2

Mathematical language

Fraction
Improper fraction
Proper fraction
Vulgar fraction
Top-heavy fraction
Percentage
Proportion

Notation
Diagonal fraction bar / horizontal fraction bar

Pedagogical notes

Describe $^1/_3$ as 'there are three equal parts and I take one', and $^3/_4$ as 'there are four equal parts and I take three'.
Be alert to pupils reinforcing misconceptions through language such as 'the bigger half'.
To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded.
NRICH: Teaching fractions with understanding
NCETM: Teaching fractions
NCETM: Departmental workshop: Fractions
NCETM: Glossary

Common approaches
All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'

Reasoning opportunities and probing questions

- Jenny says '1/10 is the same as proportion as 10% so 1/5 is the same proportion as 5%.' What do you think? Why?
- What is the same and what is different: 1/10 and 10% … 1/5 and 20%?
- Show this fraction as part of a square / rectangle / number line / …

Suggested activities

KM: Crazy cancelling, silly simplifying
NRICH: Rod fractions

Learning review
KM: 7M3 BAM Task

Possible misconceptions

- A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.
- Pupils may not make the connection that a percentage is a different way of describing a proportion
- Pupils may think that it is not possible to have a percentage greater than 100%

Proportional reasoning — *4 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- use ratio notation, including reduction to simplest form
- divide a given quantity into two parts in a given part:part or part:whole ratio

Return to overview

Possible themes

- Understand and use ratio notation
- Solve problems that involve dividing in a ratio

Possible key learning points

- Describe a comparison of measurements or objects using ratio notation a:b
- Simplify a ratio by cancelling common factors
- Divide a quantity in two parts in a given part:part ratio
- Divide a quantity in two parts in a given part:whole ratio

Prerequisites

- Find common factors of pairs of numbers
- Convert between standard metric units of measurement
- Convert between units of time
- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Solve comparison problems

Bring on the Maths+: Moving on up!
Ratio and proportion: #1

Mathematical language

Ratio
Proportion
Compare, comparison
Part
Simplify
Common factor
Cancel
Lowest terms
Unit

Notation
Ratio notation a:b for part:part or part:whole

Pedagogical notes

Note that ratio notation is first introduced in this stage.
When solving division in a ratio problems, ensure that pupils express their solution as two quantities rather than as a ratio.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Glossary

Common approaches
All pupils are explicitly taught to use the bar model as a way to represent a division in a ratio problem

Reasoning opportunities and probing questions

- Show me a set of objects that demonstrates the ratio 3:2. And another, and another …
- Convince me that the ratio 120mm:0.3m is equivalent to 2:5
- Always / Sometimes / Never: the smaller number comes first when writing a ratio
- Using Cuisenaire rods: If the red rod is 1, explain why d (dark green) is 3. Can you say the value for all the rods? (w, r, g, p, y, d, b, t, B, o). Extend this understanding of proportion by changing the unit rod e.g. if r = 1, p = ?; b = ?; o + 2B=? If B = 1; y = ? 3y = ?; o = ? o + p = ? If o + r = 6/7; t = ?

Suggested activities

KM: Division in a ratio and checking spreadsheet
KM: Maths to Infinity: FDPRP
KM: Stick on the Maths: Ratio and proportion
NRICH: Toad in the hole
NRICH: Mixing lemonade
NRICH: Food chains
NRICH: Tray bake

Possible misconceptions

- Some pupils may think that a:b always means part:part
- Some pupils may try to simplify a ratio without first ensuring that the units of each part are the same
- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts

Pattern sniffing	*3 lessons*
Key concepts (GCSE subject content statements) • generate terms of a sequence from a term-to-term rule	**The Big Picture**: Algebra progression map

Return to overview

Possible themes
- Investigate number patterns
- Explore number sequences
- Explore sequences

Possible key learning points
- Recognise simple arithmetic progressions
- Use a term-to-term rule to generate a linear sequence
- Use a term-to-term rule to generate a non-linear sequence

Prerequisites
- Know the vocabulary of sequences
- Find the next term in a linear sequence
- Find a missing term in a linear sequence
- Generate a linear sequence from its description

Bring on the Maths⁺: Moving on up!
Number and Place Value: #4, #5

Mathematical language
Pattern
Sequence
Linear
Term
Term-to-term rule
Ascending
Descending

Pedagogical notes
'Term-to-term rule' is the only new vocabulary for this unit.
Position-to-term rule, and the use of the nth term, are not developed until later stages.
NRICH: Go forth and generalise
NCETM: Algebra

Common approaches
All students are taught to describe the term-to-term rule for both numerical and non-numerical sequences

Reasoning opportunities and probing questions
- Show me a (non-)linear sequence. And another. And another.
- What's the same, what's different: 2, 5, 8, 11, 14, … and 4, 7, 10, 13, 16, …?
- Create a (non-linear/linear) sequence with a 3^{rd} term of '7'
- Always/ Sometimes /Never: The 10^{th} term of is double the 5^{th} term of the (linear) sequence
- Kenny thinks that the 20^{th} term of the sequence 5, 9, 13, 17, 21, … will be 105. Do you agree with Kenny? Explain your answer.

Suggested activities
KM: Maths to Infinity: Sequences
KM: Growing patterns
NRICH: Shifting times tables
NRICH: Odds and evens and more evens

Possible misconceptions
- When describing a number sequence some students may not appreciate the fact that the starting number is required as well as a term-to-term rule
- Some pupils may think that all sequences are ascending
- Some pupils may think the $(2n)^{th}$ term of a sequence is double the n^{th} term of a (linear) sequence

Measuring space — 7 lessons

Key concepts (GCSE subject content statements)

The Big Picture: [Measurement and mensuration progression map]

- use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
- use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- change freely between related standard units (e.g. time, length, area, volume/capacity, mass) in numerical contexts
- measure line segments and angles in geometric figures

Return to overview

Possible themes

- Measure accurately
- Convert between measures
- Solve problems involving measurement

Possible key learning points

- Use a ruler to accurately measure line segments to the nearest millimetre
- Use a protractor to accurately measure angles to the nearest degree
- Convert fluently between metric units of length
- Convert fluently between metric units of mass
- Convert fluently between metric units of volume / capacity
- Convert fluently between units of time
- Convert fluently between units of money

Prerequisites

- Convert between metric units
- Use decimal notation up to three decimal places when converting metric units
- Convert between common Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons
- Convert between units of time
- Use 12- and 24-hour clocks, both analogue and digital

Bring on the Maths+: Moving on up!
Measures: #3

Mathematical language

Length, distance
Mass, weight
Volume
Capacity
Metre, centimetre, millimetre
Tonne, kilogram, gram, milligram
Litre, millilitre
Hour, minute, second
Inch, foot, yard
Pound, ounce
Pint, gallon
Line segment

Notation
Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml
Abbreviations of units in the Imperial system: lb, oz

Pedagogical notes

Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.
The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin.
The prefix 'kilo-' means one thousand. This is Greek in origin.
Classify/Estimate angle first
NCETM: [Glossary]

Common approaches
Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)

Reasoning opportunities and probing questions

- Show me another way of describing 2.5km. And another. And another.
- Show me another way of describing 3.4 litres. And another. And another.
- Show me another way of describing3.7kg. And another. And another.
- Kenny thinks that 14:30 is the same time as 2.30 p.m. Do you agree with Kenny? Explain your answer.
- What's the same, what's different: 2 hours 30 minutes, 2.5 hours, 2⅓ hours and 2 hours 20 minutes?

Suggested activities

KM: [Sorting units]
KM: [Another length]
KM: [Measuring space]
KM: [Another capacity]
KM: [Stick on the Maths: Units]
NRICH: [Temperature]

Possible misconceptions

- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock
- Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than 120°.

Investigating angles — *3 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: [Position and direction progression map]

- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles

Return to overview

Possible themes

- Investigate angles

Bring on the Maths+: Moving on up!
Properties of shapes: #5

Possible key learning points

- Recognise and solve problems using vertically opposite angles
- Recognise and solve problems using angles at a point
- Recognise and solve problems using angles at a point on a line

Prerequisites

- Identify angles that meet at a point
- Identify angles that meet at a point on a line
- Identify vertically opposite angles
- Know that vertically opposite angles are equal

Mathematical language

Angle
Degrees
Right angle
Acute angle
Obtuse angle
Reflex angle
Protractor
Vertically opposite
Geometry, geometrical

Notation
Right angle notation
Arc notation for all other angles
The degree symbol (°)

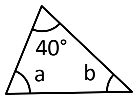
Pedagogical notes

It is important to make the connection between the total of the angles in a triangle and the sum of angles on a straight line by encouraging pupils to draw any triangle, rip off the corners of triangles and fitting them together on a straight line. However, this is not a proof and this needs to be revisited in Stage 8 using alternate angles to prove the sum is always 180°.
The word 'isosceles' means 'equal legs'. What do you have at the bottom of equal legs? Equal ankles!
NCETM: [Glossary]

Common approaches
Teachers convince pupils that the sum of the angles in a triangle is 180° by ripping the corners of triangles and fitting them together on a straight line.

Reasoning opportunities and probing questions

- Show me possible values for a and b.
  And another. And another.



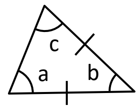
- Convince me that the angles in a triangle total 180°
- Convince me that the angles in a quadrilateral must total 360°
- What's the same, what's different: Vertically opposite angles, angles at a point, angles on a straight line and angles in a triangle?
- Kenny thinks that a triangle cannot have two obtuse angles. Do you agree? Explain your answer.
- Jenny thinks that the largest angle in a triangle is a right angle? Do you agree? Explain your thinking.

Suggested activities

KM: [Maths to Infinity: Lines and angles]
KM: [Stick on the Maths: Angles]
NRICH: [Triangle problem]
NRICH: [Square problem]
NRICH: [Two triangle problem]

Possible misconceptions

- Some pupils may think it's the 'base' angles of an isosceles that are always equal. For example, they may think that a = b rather than a = c.



- Some pupils may make conceptual mistakes when adding and subtracting mentally. For example, they may see that one of two angles on a straight line is 127° and quickly respond that the other angle must be 63°.

Calculating fractions, decimals and percentages — 15 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Fractions, decimals and percentages progression map

- apply the four operations, including formal written methods, to simple fractions (proper and improper), and mixed numbers
- interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively
- compare two quantities using percentages
- solve problems involving percentage change, including percentage increase/decrease

Return to overview

Possible themes

- Calculate with fractions
- Calculate with percentages

Possible key learning points

- Add proper and improper fractions
- Add mixed numbers
- Subtract proper and improper fractions
- Subtract mixed numbers
- Multiply proper and improper fractions
- Multiply mixed numbers
- Divide a proper fraction by a proper fraction
- Divide improper fractions
- Divide a mixed number by a proper fraction/mixed number
- Identify the multiplier for a percentage increase or decrease
- Use calculators to find a percentage of an amount using multiplicative methods
- Use calculators to increase and decrease an amount by a percentage using multiplicative methods
- Compare two quantities using percentages
- Know that percentage change = actual change ÷ original amount
- Calculate the percentage change in a given situation, including percentage increase / decrease

Prerequisites

- Add and subtract fractions with different denominators
- Add and subtract mixed numbers with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Simplify the answer to a calculation when appropriate
- Use non-calculator methods to find a percentage of an amount
- Convert between fractions, decimals and percentages

Bring on the Maths⁺: Moving on up!
Fractions, decimals & percentages: #3, #4, #5
Ratio and proportion: #2

Mathematical language

Mixed number
Equivalent fraction
Simplify, cancel, lowest terms
Proper fraction, improper fraction, top-heavy fraction, vulgar fraction
Percent, percentage
Multiplier
Increase, decrease

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Pedagogical notes

It is important that pupils are clear that the methods for addition and subtraction of fractions are different to the methods for multiplication and subtraction. A fraction wall is useful to help visualise and re-present the calculations.
NCETM: The Bar Model, Teaching fractions, Fractions videos
NCETM: Glossary

Common approaches
When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy
Teachers use the horizontal fraction bar notation at all times

Reasoning opportunities and probing questions

- Show me a proper (improper) fraction. And another. And another.
- Show me a mixed number fraction. And another. And another.
- Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain your answer.
- Benny thinks that you can only divide fractions if they have the same common denominator. Do you agree with Jenny? Explain.
- Kenny thinks that $\frac{6}{10} \div \frac{3}{2} = \frac{2}{5}$.Do you agree with Kenny? Explain.
- Always/Sometimes/Never: To reverse an increase of x%, you decrease by x%
- Lenny calculates the % increase of £6 to £8 as 25%. Do you agree with Lenny? Explain your answer.

Suggested activities

KM: Stick on the Maths: Percentage increases and decreases
KM: Maths to Infinity: FDPRP
KM: Percentage methods
KM: Mixed numbers: mixed approaches
NRICH: Would you rather?
NRICH: Keep it simple
NRICH: Egyptian fractions
NRICH: The greedy algorithm
NRICH: Fractions jigsaw
NRICH: Countdpwn fractions

Learning review
KM: 7M4 BAM Task, 7M5 BAM Task

Possible misconceptions

- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think the multiplier for, say, a 20% decrease is 0.2 rather than 0.8
- Some pupils may think that percentage change = actual change ÷ new amount

Solving equations and inequalities	*5 lessons*

Key concepts (GCSE subject content statements) **The Big Picture**: Algebra progression map

- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)
- solve linear equations in one unknown algebraically

Return to overview

Possible themes

- Explore way of solving equations
- Solve two-step equations
- Solve three-step equations

Possible key learning points

- Solve one-step equations when the solution is a positive integer or fraction
- Solve two-step equations when the solution is a positive integer or fraction
- Solve three-step equations when the solution is a positive integer or fraction
- Solve multi-step equations including the use of brackets when the solution is a positive integer or fraction
- Solve equations when the solution is an integer or fraction

Prerequisites

- Know the basic rules of algebraic notation
- Express missing number problems algebraically
- Solve missing number problems expressed algebraically

Bring on the Maths+: Moving on up!
Algebra: #2

Mathematical language

Algebra, algebraic, algebraically
Unknown
Equation
Operation
Solve
Solution
Brackets
Symbol
Substitute

Notation
The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. 2a is used rather than a2.
Division is written as a fraction

Pedagogical notes

This unit focuses on solving linear equations with unknowns on one side. Although linear equations with the unknown on both sides are addressed in Stage 8, pupils should be encouraged to think how to solve these equations by exploring the equivalent family of equations such as if 2x = 8 then 2x + 2 = 10, 2x – 3 = 5, 3x = x + 8,
3x + 2 = x + 10, etc.
Encourage pupils to re-present the equations such as 2x + 8 = 23 using the Bar Model.

x	x	8
23		

x	x
15	

x
7.5

NCETM: The Bar Model
NCETM: Algebra,
NCETM: Glossary

Common approaches
Pupils could explore solving equations by applying inverse operations, but the expectation is that all pupils should solve by balancing:

2x + 8	*=*	*23*
- 8		*- 8*
2x	*=*	*15*
÷ 2		*2*
x	*=*	*7.5 (or $^{15}/_{2}$)*

Pupils are expected to multiply out the brackets before solving an equation involving brackets. This makes the connection with two step equations such as 2x + 6 = 22

Reasoning opportunities and probing questions

- Show me an (one-step, two-step) equation with a solution of 14 (positive, fractional solution). And another. And another …
- Kenny thinks if 6x = 3 then x = 2. Do you agree with Kenny? Explain
- Jenny and Lenny are solving: 3(x – 2) = 51. Who is correct? Explain

Jenny's solution			***Lenny's solution***		
3(x – 2)	*=*	*15*	*3(x – 2)*	*=*	*15*
÷ 3		*÷ 3*	*Multiplying*	*out*	*brackets*
x - 2	*=*	*5*	*3x - 6*	*=*	*15*
÷ 2		*÷ 2*	*+2*		*+2*
x	*=*	*7*	*3x*	*=*	*21*
			÷ 3		*÷ 3*
			x =	*=*	*7*

Suggested activities

KM: Balancing: Act I
KM: Balancing: Act II
KM: Balancing: Act III
KM: Spiders and snakes. The example is for an unknown on both sides but the same idea can be used.
NRICH: Inspector Remorse
NRICH: Quince, quonce, quance
NRICH: Weighing the baby

Learning review
KM: 7M10 BAM Task

Possible misconceptions

- Some pupils may think that equations always need to be presented in the form ax + b = c rather than c = ax + b.
- Some pupils may think that the solution to an equation is always positive and/or a whole number.
- Some pupils may get the use the inverse operations in the wrong order, for example, to solve 2x + 18 = 38 the pupils divide by 2 first and then subtract 18.

Calculating space — 5 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- use standard units of measure and related concepts (length, area, volume/capacity)
- calculate perimeters of 2D shapes
- know and apply formulae to calculate area of triangles, parallelograms, trapezia
- *calculate surface area of cuboids*
- know and apply formulae to calculate volume of cuboids
- understand and use standard mathematical formulae

Return to overview

Possible themes

- Develop knowledge of area
- Investigate surface area
- Explore volume

Possible key learning points

- Calculate perimeters of 2D shapes
- Use and apply the formula to calculate the area of triangles
- Use and apply the formula to calculate the area of trapezia
- Use and apply the formula to calculate the volume of cuboids
- Find the surface area of cuboids (including cubes)

Prerequisites

- Understand the meaning of area, perimeter, volume and capacity
- Know how to calculate areas of rectangles, parallelograms and triangles using the standard formulae
- Know that the area of a triangle is given by the formula area = ½ × base × height = base × height ÷ 2 = $\frac{bh}{2}$
- Know appropriate metric units for measuring area and volume

Bring on the Maths⁺: Moving on up!
Measures: #4, #5, #6

Mathematical language

Perimeter, area, volume, capacity, surface area
Square, rectangle, parallelogram, triangle, trapezium (trapezia)
Polygon
Cube, cuboid
Square millimetre, square centimetre, square metre, square kilometre
Cubic centimetre, centimetre cube
Formula, formulae
Length, breadth, depth, height, width

Notation
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

Ensure that pupils make connections with the area and volume work in Stage 6 and below, in particular the importance of the perpendicular height.
NCETM: Glossary

Common approaches

Pupils have already derived the formula for the area of a parallelogram. They use this to derive the formula for the area of a trapezium as $\frac{(a+b)h}{2}$ *by copying and rotating a trapezium as shown above.*
Pupils use the area of a triangle as given by the formula area = $\frac{bh}{2}$.
Every classroom has a set of area posters on the wall.

Reasoning opportunities and probing questions

- Always / Sometimes / Never: The value of the volume of a cuboid is greater than the value of the surface area
- Convince me that the area of a triangle = ½ × base × height = base × height ÷ 2 = $\frac{bh}{2}$
- (Given a right-angled trapezium with base labelled 8 cm, height 5 cm, top 6 cm) Kenny uses the formula for the area of a trapezium and Benny splits the shape into a rectangle and a triangle. What would you do? Why?

Suggested activities

KM: Perimeter
KM: Triangles
KM: Equable shapes (for both 2D and 3D shapes)
KM: Triangle takeaway
KM: Surface area
KM: Class of rice
KM: Stick on the Maths: Area and Volume
KM: Maths to Infinity: Area and Volume
NRICH: Can They Be Equal?

Learning review
KM: 7M12 BAM Task

Possible misconceptions

- Some pupils may use the sloping height when finding the areas of parallelograms, triangles and trapezia
- Some pupils may think that the area of a triangle is found using area = base × height
- Some pupils may think that you multiply all the numbers to find the area of a shape
- Some pupils may confuse the concepts of surface area and volume
- Some pupils may only find the area of the three 'distinct' faces when finding surface area

Mathematical movement — 7 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Position and direction progression map

- work with coordinates in all four quadrants
- *understand and use lines parallel to the axes, y = x and y = -x*
- solve geometrical problems on coordinate axes
- identify, describe and construct congruent shapes including on coordinate axes, by considering rotation, reflection and translation
- describe translations as 2D vectors

Return to overview

Possible themes

- Explore lines on the coordinate grid
- Use transformations to move shapes
- Describe transformations

Possible key learning points

- Solve geometrical problems on coordinate axes
- Write the equation of a line parallel to the x-axis or the y-axis
- Identify and draw the lines y = x and y = -x
- Construct and describe reflections in horizontal, vertical and diagonal mirror lines (45° from horizontal)
- Describe a translation as a 2D vector
- Construct and describe rotations using a given angle, direction and centre of rotation
- Solve problems involving rotations, reflections and translations

Prerequisites

- Work with coordinates in all four quadrants
- Carry out a reflection in a given vertical or horizontal mirror line
- Carry out a translation

Bring on the Maths⁺: Moving on up!
Position and direction: #1, #2

Mathematical language

(Cartesian) coordinates
Axis, axes, x-axis, y-axis
Origin
Quadrant
Translation, Reflection, Rotation
Transformation
Object, Image
Congruent, congruence
Mirror line
Vector
Centre of rotation

Notation
Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y)
Vector notation $\binom{a}{b}$ where a = movement right and b = movement up

Pedagogical notes

Pupils should be able to use a centre of rotation that is outside, inside, or on the edge of the object
Pupils should be encouraged to see the line x = a as the complete (and infinite) set of points such that the x-coordinate is a.
The French mathematician Rene Descartes introduced Cartesian coordinates in the 17^{th} century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.
NCETM: Glossary

Common approaches
Pupils use ICT to explore these transformations
Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!'
Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.

Reasoning opportunities and probing questions

- Always / Sometimes / Never: The centre of rotation is in the centre of the object
- Convince me that y = 0 is the x-axis
- Always / Sometimes / Never: The line x = a is parallel to the x-axis

Suggested activities

KM: Lines
KM: Moving house
KM: Transformations: Bop It?
KM: Dynamic Autograph files: Reflection, Rotation, Translation
KM: Autograph transformations
KM: Stick on the Maths SSM7: Transformations
NRICH: Transformation Game

Learning review
KM: 7M11 BAM Task

Possible misconceptions

- Some pupils will wrestle with the idea that a line x = a is parallel to the y-axis
- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a shape in a diagonal mirror line some students may draw a translation
- Some pupils may think that the centre of rotation is always in the centre of the shape
- Some pupils will confuse the order of x- and y-coordinates
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Presentation of data — *6 lessons*

Key concepts (GCSE subject content statements) **The Big Picture**: Statistics progression map

- interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use

Return to overview

Possible themes

- Explore types of data
- Construct and interpret graphs
- Select appropriate graphs and charts

Possible key learning points

- Interpret and construct frequency tables
- Construct and interpret bar charts and know their appropriate use
- Construct and interpret comparative bar charts
- Construct and interpret pie charts and know their appropriate use
- Construct and interpret vertical line charts
- Choose appropriate graphs or charts to represent data

Prerequisites

- Construct and interpret a pictogram
- Construct and interpret a bar chart
- Construct and interpret a line graph
- Understand that pie charts are used to show proportions
- Use a template to construct a pie chart by scaling frequencies

Bring on the Maths+: Moving on up!
Statistics: #1, #2, #3

Mathematical language

Data, Categorical data, Discrete data
Pictogram, Symbol, Key
Frequency
Table, Frequency table
Tally
Bar chart
Time graph, Time series
Bar-line graph, Vertical line chart
Scale, Graph
Axis, axes
Line graph
Pie chart
Sector
Angle
Maximum, minimum

Notation
When tallying, groups of five are created by striking through each group of four

Pedagogical notes

In stage 6 pupils constructed pie charts when the total of frequencies is a factor of 360. More complex cases can now be introduced.
Much of the content of this unit has been covered previously in different stages. This is an opportunity to bring together the full range of skills encountered up to this point, and to develop a more refined understanding of usage and vocabulary.

William Playfair, a Scottish engineer and economist, introduced the bar chart and line graph in 1786. He also introduced the pie chart in 1801.
NCETM: Glossary

Common approaches
Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency and not using percentages.
The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.

Reasoning opportunities and probing questions

- Show me a pie chart representing the following information: Blue (30%), Red (50%), Yellow (the rest). And another. And another.
- Always / Sometimes / Never: Bar charts are vertical
- Always / Sometimes / Never: Bar charts, pie charts, pictograms and vertical line charts can be used to represent any data
- Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer.

Suggested activities

KM: Constructing pie charts
KM: Maths to Infinity: Averages, Charts and Tables
NRICH: Picturing the World
NRICH: Charting Success

Possible misconceptions

- Some pupils may think that the lines on a line graph are always meaningful
- Some pupils may think that each square on the grid used represents one unit
- Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°
- Some pupils may not leave gaps between the bars of a bar chart

Measuring data — 7 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range)

Return to overview

Possible themes

- Investigate averages
- Explore ways of summarising data
- Analyse and compare sets of data

Possible key learning points

- Find the mode of set of data
- Find the median of a set of data including when there are an even number of numbers in the data set
- Calculate the mean from a frequency table
- Find the mode from a frequency table
- Find the median from a frequency table
- Calculate and understand the range as a measure of spread (or consistency)
- Analyse and compare sets of data, appreciating the limitations of different statistics (mean, median, mode, range)

Prerequisites

- Understand the meaning of 'average' as a typicality (or location)
- Calculate the mean of a set of data

Bring on the Maths⁺: Moving on up!
Statistics: #4

Mathematical language

Average
Spread
Consistency
Mean
Median
Mode
Range
Measure
Data
Statistic
Statistics
Approximate
Round

Pedagogical notes

The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean').
NCETM: Glossary

Common approaches
Every classroom has a set of statistics posters on the wall
Always use brackets when writing out the calculation for a mean, e.g. (2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5

Reasoning opportunities and probing questions

- Show me a set of data with a mean (mode, median, range) of 5.
- Always / Sometimes / Never: The mean is greater than the mode for a set of data
- Always / Sometimes / Never: The mean is greater than the median for a set of data
- Convince me that a set of data could have more than one mode.
- What's the same and what's different: mean, mode, median, range?

Suggested activities

KM: Maths to Infinity: Averages
KM: Maths to Infinity: Averages, Charts and Tables
KM: Stick on the Maths HD4: Averages
NRICH: M, M and M
NRICH: The Wisdom of the Crowd

Possible misconceptions

- If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as 2 + 3 + 4 + 5 ÷ 4 = 10.25.
- Some pupils may think that the range is a type of average
- Some pupils may think that a set of data with an even number of items has two values for the median, e.g. 2, 4, 5, 6, 7, 8 has a median of 5 and 6 rather than 5.5
- Some pupils may not write the data in order before finding the median.