Numbers and the number system — *7 lessons*

Key concepts (GCSE subject content statements) — **The Big Picture**: Number and Place Value progression map

- use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Return to overview

Possible themes

- Identify and use the prime factorisation of a number
- Understand and use standard form

Possible key learning points

- Write a number as a product of its prime factors
- Use prime factorisations to find the highest common factor of two numbers
- Use prime factorisations to find the lowest common multiple of two numbers
- Solve problems using highest common factors or lowest common multiples
- Round numbers to a given number of significant figures
- Use standard form to write large numbers
- Use standard form to write small numbers

Prerequisites

- Know the meaning of a prime number
- Recall prime numbers up to 50
- Understand the use of notation for powers
- Know how to round to the nearest whole number, 10, 100, 1000 and to decimal places
- Multiply and divide numbers by powers of 10
- Know how to identify the first significant figure in any number
- Approximate by rounding to the first significant figure in any number

Mathematical language

Prime
Prime factor
Prime factorisation
Product
Venn diagram
Highest common factor
Lowest common multiple
Standard form
Significant figure

Notation
Index notation: e.g. 5^3 is read as '5 to the power of 3' and means '3 lots of 5 multiplied together'
Standard form (see Key concepts) is sometimes called 'standard index form', or more properly, 'scientific notation'

Pedagogical notes

Pupils should explore the ways to enter and interpret numbers in standard form on a scientific calculator. Different calculators may very well have different displays, notations and methods.
Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
NRICH: Divisibility testing
NCETM: Glossary

Common approaches
The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'

Reasoning opportunities and probing questions

- Show me two (three-digit) numbers with a highest common factor of 18. And another. And another…
- Show me two numbers with a lowest common multiple of 240. And another. And another…
- Jenny writes $7.1 \times 10^{-5} = 0.0000071$. Kenny writes $7.1 \times 10^{-5} = 0.000071$. Who do you agree with? Give reasons for your answer.

Suggested activities

Use the number 5040 when writing prime factorisations
KM: Ben Nevis
KM: Astronomical numbers
KM: Interesting standard form
KM: Powers of ten
KM: Maths to Infinity: Standard form
Powers of ten film (external site)
The scale of the universe animation (external site)

Learning review
KM: 8M2 BAM Task

Possible misconceptions

- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'
- Some pupils may think 35 934 = 36 to two significant figures
- When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros; e.g. $4 \times 10^5 = 400\,000$ so $4.2 \times 10^5 = 4\,200\,000$
- Similarly, when working with small numbers (negative powers of 10) some pupils may think that the power indicates how many zeros should be placed between the decimal point and the first non-zero digit

Calculating

14 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Calculation progression map

- apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative
- use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

Return to overview

Possible themes

- Calculate with negative numbers
- Apply the correct order of operations

Possible key learning points

- Subtract a number from a smaller number
- Add a positive number to a negative number
- Subtract a positive number from a negative number
- Add a negative number
- Subtract a negative number
- Multiply a positive number by a negative number
- Multiply a negative number by a negative number
- Divide a positive number by a negative number
- Divide a negative number by a negative number
- Square and cube positive and negative numbers
- Use a scientific calculator to calculate with negative numbers
- Use a scientific calculator to calculate with fractions, both positive and negative
- Understand how to use the order of operations including powers
- Understand how to use the order of operations including roots

Prerequisites

- Fluently recall and apply multiplication facts up to 12 × 12
- Know and use column addition and subtraction
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Apply the four operations with fractions and mixed numbers
- Convert between an improper fraction and a mixed number
- Know the order of operations for the four operations and brackets

Bring on the Maths⁺: Moving on up!
Number and Place Value: v3

Mathematical language

Negative number
Directed number
Improper fraction
Top-heavy fraction
Mixed number
Operation
Inverse
Long multiplication
Short division
Power
Indices
Roots

Pedagogical notes

Pupils need to know how to enter negative numbers into their calculator and how to interpret the display.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
NRICH: Adding and subtracting positive and negative numbers
NRICH: History of negative numbers
NCETM: Departmental workshop: Operations with Directed Numbers
NCETM: Glossary

Common approaches
Teachers use the language 'negative number', and not 'minus number', to avoid confusion with calculations
Every classroom has a negative number washing line on the wall
Long multiplication and short division are to be promoted as the 'most efficient methods'.
If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.

Reasoning opportunities and probing questions

- Convince me that -3 - -7 = 4
- Show me an example of a calculation involving addition of two negative numbers and the solution -10. And another. And another …
- Create a Carroll diagram with 'addition', 'subtraction' as the column headings and 'one negative number', 'two negative numbers' as the row headings. Ask pupils to create (if possible) a calculation that can be placed in each of the four positions. If they think it is not possible, explain why. Repeat for multiplication and division.

Suggested activities

KM: Summing up
KM: Developing negatives
KM: Sorting calculations
KM: Maths to Infinity: Directed numbers
Standards Unit: N9 Evaluating directed number statements
NRICH: Working with directed numbers

Learning review
KM: 8M1 BAM Task

Possible misconceptions

- Some pupils may use a rule stated as 'two minuses make a plus' and make many mistakes as a result; e.g. -4 + -6 = 10
- Some pupils may incorrectly apply the principle of commutativity to subtraction; e.g. 4 – 7 = 3
- The order of operations is often not applied correctly when squaring negative numbers. As a result pupils may think that $x^2 = -9$ when $x = -3$. The fact that a calculator applies the correct order means that $-3^2 = -9$ and this can actually reinforce the misconception. In this situation brackets should be used as follows: $(-3)^2 = 9$.

Visualising and constructing — 9 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Properties of Shape progression map

- measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement
- interpret plans and elevations of 3D shapes
- use scale factors, scale diagrams and maps

Return to overview

Possible themes

- Explore enlargement of 2D shapes
- Use and interpret scale drawings
- Use and interpret bearings
- Explore ways of representing 3D shapes

Possible key learning points

- Use the centre and scale factor to carry out an enlargement with a positive integer scale factor
- Find the centre of enlargement
- Find the scale factor of an enlargement
- Use scale diagrams, including maps
- Use the concept of scaling in diagrams
- Interpret plans and elevations
- Understand and use bearings
- Construct scale diagrams involving bearings
- Solve geometrical problems using bearings

Prerequisites

- Use a protractor to measure angles to the nearest degree
- Use a ruler to measure lengths to the nearest millimetre
- Understand coordinates in all four quadrants
- Work out a multiplier given two numbers
- Understand the concept of an enlargement (no scale factor)

Mathematical language

Similar, Similarity
Enlarge, enlargement
Scaling
Scale factor
Centre of enlargement
Object
Image
Scale drawing
Bearing
Plan, Elevation

Notation

Bearings are always given as three figures; e.g. 025°.
Cartesian coordinates: separated by a comma and enclosed by brackets

Pedagogical notes

Describing enlargement as a 'scaling' will help prevent confusion when dealing with fractional scale factors
NCETM: Departmental workshops: Enlargement
NCETM: Glossary

Common approaches

All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.

Reasoning opportunities and probing questions

- Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived?
- Show me an example of a sketch where the bearing of A from B is between 90° and 180°. And another. And another …
- The bearing of A from B is 'x'. Find the bearing of B from A in terms of 'x'. Explain why this works.
- Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.

Suggested activities

KM: Outdoor Leisure 13
KM: Airports and hilltops
KM: Plans and elevations
KM: Transformation template
KM: Enlargement I
KM: Enlargement II
KM: Investigating transformations with Autograph (enlargement and Main Event II). Dynamic example.
KM: Solid problems (plans and elevations)
KM: Stick on the Maths: plans and elevations
WisWeb applet: Building houses
NRICH: Who's the fairest of them all?

Learning review

www.diagnosticquestions.com

Possible misconceptions

- Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape.
- If the bearing of A from B is 'x', then some pupils may think that the bearing of B from A is '180 – x'.
- The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

Understanding risk I — *6 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Probability progression map

- relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
- record describe and analyse the frequency of outcomes of probability experiments using tables
- construct theoretical possibility spaces for single experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply the property that the probabilities of an exhaustive set of outcomes sum to one

Return to overview

Possible themes

- Understand the meaning of probability
- Explore experiments and outcomes
- Develop understanding of probability

Possible key learning points

- Know and use the vocabulary of probability
- Understand the use of the 0-1 scale to measure probability
- List all the outcomes for an experiment, including the use of tables
- Work out theoretical probabilities for events with equally likely outcomes
- Know that the sum of probabilities for all outcomes is 1
- Apply the fact that the sum of probabilities for all outcomes is 1

Prerequisites

- Understand the equivalence between fractions, decimals and percentages
- Compare fractions, decimals or percentages
- Simplify a fraction by cancelling common factors

Mathematical language

Probability, Theoretical probability
Event
Outcome
Impossible, Unlikely, Evens chance, Likely, Certain
Equally likely
Mutually exclusive
Exhaustive
Possibility space
Experiment

Notation
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

Pedagogical notes

This is the first time students will meet probability.
It is not immediately apparent how to use words to label the middle of the probability scale. 'Evens chance' is a common way to do so, although this can be misleading as it could be argued that there is an even chance of obtaining any number when rolling a fair die.
NRICH: Introducing probability
NRICH: Why Do People Find Probability Unintuitive and Difficult?
NCETM: Glossary

Common approaches
Every classroom has a display of a probability scale labeled with words and numbers. Pupils create events and outcomes that are placed on this scale.

Reasoning opportunities and probing questions

- Show me an example of an event and outcome with a probability of 0. And another. And another…
- Always / Sometimes / Never: if I pick a card from a pack of playing cards then the probability of picking a club is ¼
- Label this (eight-sided) spinner so that the probability of scoring a 2 is ¼. How many different ways can you label it?

Suggested activities

KM: Probability scale and slideshow version
KM: Probability loop cards
NRICH: Dice and spinners interactive

Learning review
KM: 8M13 BAM Task

Possible misconceptions

- Some pupils will initially think that, for example, the probability of it raining tomorrow is ½ as it either will or it won't.
- Some students may write a probability as odds (e.g. 1:6 or '1 to 6'). There is a difference between probability and odds, and therefore probabilities must only be written as fractions, decimals or percentages.
- Some pupils may think that, for example, if they flip a fair coin three times and obtain three heads, then it must be more than likely they will obtain a head next.

Algebraic proficiency: tinkering — 10 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- use and interpret algebraic notation, including: a^2b in place of $a \times a \times b$, coefficients written as fractions rather than as decimals
- understand and use the concepts and vocabulary of factors
- simplify and manipulate algebraic expressions by taking out common factors and simplifying expressions involving sums, products and powers, including the laws of indices
- substitute numerical values into scientific formulae
- rearrange formulae to change the subject

Return to overview

Possible themes

- Understand the concept of a factor
- Understand the notation of algebra
- Manipulate algebraic expressions
- Evaluate algebraic statements

Possible key learning points

- Use and interpret algebraic notation, including: $a^2\,b$ in place of $a \times a \times b$, coefficients written as fractions rather than as decimals
- Simplify an expression involving terms with combinations of variables (e.g. $3a^2b + 4ab^2 + 2a^2 - a^2b$)
- Factorise an algebraic expression by taking out common factors
- Simplify expressions using the law of indices for multiplication
- Simplify expressions using the law of indices for division
- Simplify expressions using the law of indices for powers
- Know and use the zero index
- Substitute positive and negative numbers into formulae
- Change the subject of a formula when one step is required
- Change the subject of a formula when two steps are required

Prerequisites

- Know basic algebraic notation (the rules of algebra)
- Simplify an expression by collecting like terms
- Know how to multiply a single term over a bracket
- Substitute positive numbers into expressions and formulae
- Calculate with negative numbers

Mathematical language

Product
Variable
Term
Coefficient
Common factor
Factorise
Power
Indices
Formula, Formulae
Subject
Change the subject

Notation
See Key concepts above

Pedagogical notes

During this unit pupils should experience factorising a quadratic expression such as $6x^2 + 2x$.
Collaborate with the science department to establish a list of formulae that will be used, and ensure consistency of approach and experience.
NCETM: Algebra
NCETM: Departmental workshop: Index Numbers
NCETM: Departmental workshops: Deriving and Rearranging Formulae
NCETM: Glossary

Common approaches
Once the laws of indices have been established, all teachers refer to 'like numbers multiplied, add the indices' and 'like numbers divided, subtract the indices. They also generalise to $a^m \times a^n = a^{m+n}$, etc.
When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a 'change side, change sign' approach.

Reasoning opportunities and probing questions

- Convince me $a^0 = 1$.
- What is wrong with this statement and how can it be corrected: $5^2 \times 5^4 = 5^8$?
- Jenny thinks that if $y = 2x + 1$ then $x = (y - 1)/2$. Kenny thinks that if $y = 2x + 1$ then $x = y/2 - 1$. Who do you agree with? Explain your thinking.

Suggested activities

KM: Missing powers
KM: Laws of indices. Some useful questions.
KM: Maths to Infinity: Indices
KM: Scientific substitution (Note that page 2 is hard)
NRICH: Temperature

Learning review
KM: 8M3 BAM Task, 8M7 BAM Task, 8M8 BAM Task

Possible misconceptions

- Some pupils may misapply the order of operation when changing the subject of a formula
- Many pupils may think that $a^0 = 0$
- Some pupils may not consider 4ab and 3ba as 'like terms' and therefore will not 'collect' them when simplifying expressions

Exploring fractions, decimals and percentages — *5 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Fractions, decimals and percentages progression map

- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8)

Return to overview

Possible themes

- Explore links between fractions, decimals and percentages

Possible key learning points

- Identify if a fraction is terminating or recurring
- Recall some decimal and fraction equivalents (e.g. tenths, fifths, eighths, thirds, quarters, etc.)
- Write a terminating decimal as a fraction
- Write a fraction in its lowest terms by cancelling common factors
- Use a calculator to change any fraction to a decimal

Prerequisites

- Understand that fractions, decimals and percentages are different ways of representing the same proportion
- Convert between mixed numbers and top-heavy fractions
- Write one quantity as a fraction of another

Mathematical language

Fraction
Mixed number
Top-heavy fraction
Percentage
Decimal
Proportion
Terminating
Recurring
Simplify, Cancel

Notation
Diagonal and horizontal fraction bar

Pedagogical notes

The diagonal fraction bar (solidus) was first used by Thomas Twining (1718) when recorded quantities of tea. The division symbol (÷) is called an obelus, but there is no name for a horizontal fraction bar.
NRICH: History of fractions
NRICH: Teaching fractions with understanding
NCETM: Glossary

Common approaches
All pupils should use the horizontal fraction bar to avoid confusion when fractions are coefficients in algebraic situations

Reasoning opportunities and probing questions

- Without using a calculator, convince me that 3/8 = 0.375
- Show me a fraction / decimal / percentage equivalent. And another. And another …
- What is the same and what is different: 2.5, 25%, 0.025, ¼ ?

Suggested activities

KM: FDP conversion. Templates for taking notes.
KM: Fraction sort. Tasks one and two only.
KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion
NRICH: Matching fractions, decimals and percentages

Learning review
KM: 8M4 BAM Task

Possible misconceptions

- Some pupils may make incorrect links between fractions and decimals such as thinking that 1/5 = 0.15
- Some pupils may think that 5% = 0.5, 4% = 0.4, etc.
- Some pupils may think it is not possible to have a percentage greater than 100%.

Proportional reasoning — *11 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Ratio and Proportion progression map

- express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
- identify and work with fractions in ratio problems
- understand and use proportion as equality of ratios
- express a multiplicative relationship between two quantities as a ratio or a fraction
- use compound units (such as speed, rates of pay, unit pricing)
- change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts
- relate ratios to fractions and to linear functions

Return to overview

Possible themes

- Explore the uses of ratio
- Investigate the connection between ratio and proportion
- Solve problems involving proportional reasoning
- Solve problems involving compound units

Possible key learning points

- Express the division of a quantity into two parts as a ratio
- Understand the connections between ratios and fractions
- Find a relevant multiplier in a situation involving proportion
- Solve ratio problems involving mixing
- Solve ratio problems involving comparison
- Solve ratio problems involving concentrations
- Understand and use compound units
- Convert between units of speed
- Solve problems involving speed
- Solve problems involving rates of pay
- Solve problems involving unit pricing

Prerequisites

- Understand and use ratio notation
- Divide an amount in a given ratio

Mathematical language

Ratio
Proportion
Proportional
Multiplier
Speed
Unitary method
Units
Compound unit

Notation
Kilometres per hour is written as km/h or kmh^{-1}
Metres per second is written as m/s or ms^{-1}

Pedagogical notes

The Bar Model is a powerful strategy for pupils to 're-present' a problem involving ratio.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Departmental workshops: Proportional Reasoning
NCETM: Glossary

Common approaches
All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving proportion

Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in proportion. And another. And another …
- (Showing a table of values such as the one below) convince me that this information shows a proportional relationship

6	9
10	15
14	21

- Which is the faster speed: 60 km/h or 10 m/s? Explain why.

Suggested activities

KM: Proportion for real
KM: Investigating proportionality
KM: Maths to Infinity: Fractions, decimals, percentages, ratio, proportion
NRICH: In proportion
NRICH: Ratio or proportion?
NRICH: Roasting old chestnuts 3
Standards Unit: N6 Developing proportional reasoning

Learning review
KM: 8M5 BAM Task

Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- Some pupils may think that a multiplier always has to be greater than 1
- When converting between times and units, some pupils may base their working on 100 minutes = 1 hour

Pattern sniffing — *4 lessons*

Key concepts (GCSE subject content statements)

- generate terms of a sequence from either a term-to-term or a position-to-term rule
- deduce expressions to calculate the nth term of linear sequences

The Big Picture: Algebra progression map

Return to overview

Possible themes

- Explore sequences

Possible key learning points

- Generate terms of a sequence from a position-to-term rule
- Find the nth term of an ascending linear sequence
- Find the nth term of an descending linear sequence
- Use the nth term of a sequence to deduce if a given number is in a sequence

Prerequisites

- Use a term-to-term rule to generate a sequence
- Find the term-to-term rule for a sequence
- Describe a sequence using the term-to-term rule

Mathematical language

Sequence
Linear
Term
Difference
Term-to-term rule
Position-to-term rule
Ascending
Descending

Notation
T(n) is often used when finding the nth term of sequence

Pedagogical notes

Using the nth term for times tables is a powerful way of finding the nth term for any linear sequence. For example, if the pupils understand the 3 times table can ne described as '3n' then the linear sequence 4, 7, 10, 13, … can be described as the 3 times table 'shifted up' one place, hence 3n + 1.
Exploring statements such as 'is 171 is in the sequence 3, 9, 15, 21, 27, ..?' is a very powerful way for pupils to realise that 'term-to-term' rules can be inefficient and therefore 'position-to-term' rules (nth term) are needed.
NCETM: Algebra
NCETM: Glossary

Common approaches
Teachers refer to a sequence such as 2, 5, 8, 11, … as 'the three times table minus one', to help pupils construct their understanding of the nth term of a sequence.
All students have the opportunity to use spreadsheets to generate sequences

Reasoning opportunities and probing questions

- Show me a sequence that could be generated using the nth term 4n ± c. And another. And another …
- What's the same, what's different: 4, 7, 10, 13, 16, …. , 2, 5, 8, 11, 14, … , 4, 9, 14, 19, 24, …. and 4, 10, 16, 22, 28, …?
- The 4th term of a linear sequence is 15. Show me the nth term of a sequence with this property. And another. And another …
- Convince me that the nth term of the sequence 2, 5, 8, 11, … is 3n -1 .
- Kenny says the 171 is in the sequence 3, 9, 15, 21, 27, … Do you agree with Kenny? Explain your reasoning.

Suggested activities

KM: Spreadsheet sequences
KM: Generating sequences
KM: Brackets and sequences
KM: Maths to Infinity: Sequences
KM: Stick on the Maths: Linear sequences
NRICH: Charlie's delightful machine
NRICH: A little light thinking
NRICH: Go forth and generalise

Learning review
KM: 8M9 BAM Task

Possible misconceptions

- Some pupils will think that the nth term of the sequence 2, 5, 8, 11, … is n + 3.
- Some pupils may think that the (2n)th term is double the nth term of a linear sequence.
- Some pupils may think that sequences with nth term of the form 'ax ± b' must start with 'a'.

Investigating angles — *7 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Position and direction progression map

- understand and use alternate and corresponding angles on parallel lines
- derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

Return to overview

Possible themes

- Develop knowledge of angles
- Explore geometrical situations involving parallel lines

Possible key learning points

- Solve missing angle problems involving alternate angles
- Solve missing angle problems involving corresponding angles
- Use knowledge of alternate and corresponding angles to calculate missing angles in geometrical diagrams
- Establish the fact that angles in a triangle must total 180°
- Establish the size of an interior angle in a regular polygon
- Establish the size of an exterior angle in a regular polygon
- Solve missing angle problems in polygons

Prerequisites

- Use angles at a point, angles at a point on a line and vertically opposite angles to calculate missing angles in geometrical diagrams
- Know that the angles in a triangle total 180°

Mathematical language

Degrees
Right angle, acute angle, obtuse angle, reflex angle
Vertically opposite
Geometry, geometrical
Parallel
Alternate angles, corresponding angles
Interior angle, exterior angle
Regular polygon

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Arrow notation to show parallel lines

Pedagogical notes

The KM: Perplexing parallels resource is a great way for pupils to discover practically the facts for alternate and corresponding angles.
Pupils have established the fact that angles in a triangle total 180° in Stage 7. However, using alternate angles they are now able to prove this fact.
Encourage pupils to draw regular and irregular convex polygons to discover the sum of the interior angles = $(n - 2) \times 180°$.
NCETM: Glossary

Common approaches
Teachers insist on correct mathematical language (and not F-angles or Z-angles for example)

Reasoning opportunities and probing questions

- Show me a pair of alternate (corresponding) angles. And another. And another …
- Jenny thinks that hexagons are the only polygon that tessellates. Do you agree? Explain your reasoning.
- Convince me that the angles in a triangle total 180°.
- Convince me that the interior angle of a pentagon is 540°.
- Always/ Sometimes/ Never: The sum of the interior angles of an n-sided polygon can be calculated using sum = $(n - 2) \times 180°$.
- Always/ Sometimes/ Never: The sum of the exterior angles of a polygon is 360°.

Suggested activities

KM: Alternate and corresponding angles
KM: Perplexing parallels
KM: Investigating polygons
KM: Maths to Infinity: Lines and angles
KM: Stick on the Maths: Alternate and corresponding angles
KM: Stick on the Maths: Geometrical problems
NRICH: Ratty

Possible misconceptions

- Some pupils may think that alternate and/or corresponding angles have a total of 180° rather than being equal.
- Some pupils may think that the sum of the interior angles of an n-sided polygon can be calculated using Sum = $n \times 180°$.
- Some pupils may think that the sum of the exterior angles increases as the number of sides of the polygon increases.

Calculating fractions, decimals and percentages — *6 lessons*

Key concepts (GCSE subject content statements)

- interpret fractions and percentages as operators
- work with percentages greater than 100%
- solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics
- calculate exactly with fractions

The Big Picture: Fractions, decimals and percentages progression map

Return to overview

Possible themes

- Calculate with fractions
- Calculate with percentages

Possible key learning points

- Identify the multiplier for a percentage increase or decrease when the percentage is greater than 100%
- Use calculators to increase an amount by a percentage greater than 100%
- Solve problems involving percentage change
- Solve original value problems when working with percentages
- Solve financial problems including simple interest
- Solve problems that require exact calculation with fractions

Prerequisites

- Apply the four operations to proper fractions, improper fractions and mixed numbers
- Use calculators to find a percentage of an amount using multiplicative methods
- Identify the multiplier for a percentage increase or decrease
- Use calculators to increase (decrease) an amount by a percentage using multiplicative methods
- Know that percentage change = actual change ÷ original amount

Mathematical language

Proper fraction, improper fraction, mixed number
Simplify, cancel, lowest terms
Percent, percentage
Percentage change
Original amount
Multiplier
(Simple) interest
Exact

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Pedagogical notes

The bar model is a powerful strategy for pupils to 're-present' a problem involving percentage change.
Only simple interest should be explored in this unit. Compound interest will be developed later.
NCETM: The Bar Model
NCETM: Glossary

Common approaches
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy
Teachers use the horizontal fraction bar notation at all times

Reasoning opportunities and probing questions

- Convince me that the multiplier for a 150% increase is 2.5
- Kenny buys a poncho in a 25% sale. The sale price is £40. Kenny thinks that the original is £50. Do you agree with Kenny? Explain your answer.
- Jenny thinks that increasing an amount by 200% is the same as multiplying by 3. Do you agree with Jenny? Explain your answer.

Suggested activities

KM: Stick on the Maths: Proportional reasoning
KM: Stick on the Maths: Multiplicative methods
KM: Percentage identifying
NRICH: One or both
NRICH: Antiques roadshow

Learning review
KM: 8M6 BAM Task

Possible misconceptions

- Some pupils may think that the multiplier for a 150% increase is 1.5
- Some pupils may think that increasing an amount by 200% is the same as doubling.
- In isolation, pupils may be able to solve original value problems confidently. However, when it is necessary to identify the type of percentage problem, many pupils will apply a method for a more simple percentage increase / decrease problem. If pupils use models (e.g. the bar model, or proportion tables) to represent all problems then they are less likely to make errors in identifying the type of problem.

Solving equations and inequalities — 6 lessons

Key concepts (GCSE subject content statements)

- solve linear equations with the unknown on both sides of the equation
- find approximate solutions to linear equations using a graph

The Big Picture: Algebra progression map

Return to overview

Possible themes

- Solve linear equations with the unknown on one side
- Solve linear equations with the unknown on both sides
- Explore connections between graphs and equations

Possible key learning points

- Solve linear equations with the unknown on one side when calculating with negative numbers is required
- Solve linear equations with the unknown on both sides when the solution is a whole number
- Solve linear equations with the unknown on both sides when the solution is a fraction
- Solve linear equations with the unknown on both sides when the solution is a negative number
- Solve linear equations with the unknown on both sides when the equation involves brackets
- Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation

Prerequisites

- Choose the required inverse operation when solving an equation
- Solve linear equations by balancing when the solution is a whole number or a fraction

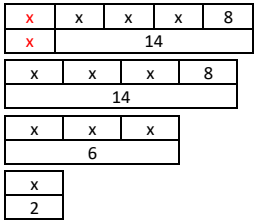
Mathematical language

Algebra, algebraic, algebraically
Unknown
Equation
Operation
Solve
Solution
Brackets
Symbol
Substitute
Graph
Point of intersection

Notation
The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. 2a is used rather than a2.
Division is written as a fraction

Pedagogical notes

This unit builds on the wok solving linear equations with unknowns on one side in Stage 7. It is essential that pupils are secure with solving these equations before moving onto unknowns on both sides.
Encourage pupils to 're-present' the problem using the Bar Model.
NCETM: The Bar Model
NCETM: Algebra
NCETM: Glossary



Common approaches
All pupils should solve equations by balancing:

$4x + 8 = 14 + x$
$-x \quad -x$
$3x + 8 = 14$
$-8 \quad -8$
$3x = 6$
$\div 3 \quad \div 3$
$x = 2$

Reasoning opportunities and probing questions

- Show me an (one-step, two-step) equation with a solution of -8 (negative, fractional solution). And another. And another …
- Show me a two-step equation that is 'easy' to solve. And another. And another …
- What's the same, what's different: 2x + 7 = 25, 3x + 7 = x + 25, x + 7 = 7 – x, 4x + 14 = 50 ?
- Convince me how you could use graphs to find solutions, or estimates, for equations.

Suggested activities

KM: Solving equations
KM: Stick on the Maths: Constructing and solving equations
NRICH: Think of Two Numbers

Learning review
KM: 8M10 BAM Task

Possible misconceptions

- Some pupils may think that you always have to manipulate the equation to have the unknowns on the LHS of the equal sign, for example 2x – 3 = 6x + 6
- Some pupils think if 4x = 2 then x = 2.
- When solving equations of the form 2x – 8 = 4 – x, some pupils may subtract 'x' from both sides.

Calculating space

8 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Measurement and mensuration progression map

- compare lengths, areas and volumes using ratio notation
- calculate perimeters of 2D shapes, including circles
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference
- know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2
- calculate areas of circles and composite shapes
- know and apply formulae to calculate volume of right prisms (including cylinders)

Return to overview

Possible themes

- Investigate circles
- Discover pi
- Solve problems involving circles
- Explore prisms and cylinders

Possible key learning points

- Know circle definitions and properties, including: centre, radius, chord, diameter, circumference
- Calculate the circumference of a circle when radius or diameter is given
- Calculate the perimeter of composite shapes that include sections of a circle
- Calculate the area of a circle when radius or diameter is given
- Calculate the area of composite shapes that include sections of a circle
- Calculate the volume of a right prism
- Calculate the volume of a cylinder
- Compare lengths, areas and volumes using ratio notation

Prerequisites

- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle
Centre
Radius, diameter, chord, circumference
Pi
(Right) prism
Cross-section
Cylinder
Polygon, polygonal
Solid

Notation
π
Abbreviations of units: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

C = πd can be established by investigating the ratio of the circumference to the diameter of circular objects (wheel, clock, tins, glue sticks, etc.) Pupils need to understand this formula in order to derive $A = \pi r^2$.
A prism is a solid with constant polygonal cross-section. A right prism is a prism with a cross-section that is perpendicular to the 'length'.
NCETM: Glossary

Common approaches
The area of a circle is derived by cutting a circle into many identical sectors and approximating a parallelogram
Every classroom has a set of area posters on the wall
The formula for the volume of a prism is 'area of cross-section × length' even if the orientation of the solid suggests that height is required
Pupils use area of a trapezium = $\frac{(a+b)h}{2}$ and area of a triangle = area = $\frac{bh}{2}$

Reasoning opportunities and probing questions

- Convince me C = 2πr = πd.
- What is wrong with this statement? How can you correct it?
  The area of a circle with radius 7 cm is approximately 441 cm^2 because $(3 \times 7)^2 = 441$.
- Convince me that the area of a semi-circle = $\frac{\pi d^2}{8}$
- Name a right prism. And another. And another …
- Convince me that a cylinder is not a prism

Suggested activities

KM: Circle connections, Circle connections v2
KM: Circle circumferences, Circle problems
KM: Circumference searching
KM: Maths to Infinity: Area and Volume
KM: Stick on the Maths: Circumference and area of a circle
KM: Stick on the Maths: Right prisms
NRICH: Blue and White
NRICH: Efficient Cutting
NRICH: Cola Can
Learning review
KM: 8M12 BAM Task

Possible misconceptions

- Some pupils will work out $(\pi \times \text{radius})^2$ when finding the area of a circle
- Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some pupils may think that the area of a triangle = base × height
- Some pupils may think that you multiply all the numbers to find the volume of a prism
- Some pupils may confuse the concepts of surface area and volume

Algebraic proficiency: visualising — 11 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Algebra progression map

- plot graphs of equations that correspond to straight-line graphs in the coordinate plane
- identify and interpret gradients and intercepts of linear functions graphically
- recognise, sketch and interpret graphs of linear functions and simple quadratic functions
- plot and interpret graphs and graphs of non-standard (*piece-wise linear)* functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed

Return to overview

Possible themes

- Plot and interpret linear graphs
- Plot and quadratic graphs
- Model real situations using linear graphs

Possible key learning points

- Know that graphs of functions of the form $y = mx + c$, $x \pm y = c$ and $ax \pm by = c$ are linear
- Plot graphs of functions of the form $y = mx \pm c$
- Plot graphs of functions of the form $ax \pm by = c$
- Find the gradient of a straight line on a unit grid
- Find the y-intercept of a straight line
- Sketch linear graphs
- Distinguish between a linear and quadratic graph
- Plot graphs of quadratic functions of the form $y = x^2 \pm c$
- Sketch a simple quadratic graph
- Plot and interpret graphs of piece-wise linear functions in real contexts
- Plot and interpret distance-time graphs (speed-time graphs) including approximate solutions to kinematic problems

Prerequisites

- Use coordinates in all four quadrants
- Write the equation of a line parallel to the x-axis or the y-axis
- Draw a line parallel to the x-axis or the y-axis given its equation
- Identify the lines y = x and y = -x
- Draw the lines y = x and y = -x
- Substitute positive and negative numbers into formulae

Mathematical language

Plot
Equation (of a graph)
Function
Formula
Linear
Coordinate plane
Gradient
y-intercept
Substitute
Quadratic
Piece-wise linear
Model
Kinematic, Speed, Distance

Notation
y = mx + c

Pedagogical notes

When plotting graphs of functions of the form y = mx + c a table of values can be useful. Note that negative number inputs can cause difficulties. Pupils should be aware that the values they have found for linear functions should correspond to a straight line.
NCETM: Glossary

Common approaches
Pupils are taught to use positive numbers wherever possible to reduce potential difficulties with substitution of negative numbers
Students plot points with a 'x' and not '•'
Students draw graphs in pencil
All pupils use dynamic geometry software to explore graphs of functions

Reasoning opportunities and probing questions

- Draw a distance-time graph of your journey to school. Explain the key features.
- Show me a point on this line (e.g. y = 2x + 1). And another, and another …
- (Given an appropriate distance-time graph) convince me that Kenny is stationary between 10: 00 a.m. and 10:45 a.m.

Suggested activities

KM: Plotting graphs
KM: Matching graphs
KM: Matching graphs (easy)
KM: Autograph 1
KM: Autograph 2
KM: The hare and the tortoise

Learning review
KM: 8M11 BAM Task

Possible misconceptions

- When plotting linear graphs some pupils may draw a line segment that stops at the two most extreme points plotted
- Some pupils may think that a sketch is a very rough drawing. It should still identify key features, and look neat, but will not be drawn to scale
- Some pupils may think that a positive gradient on a distance-time graph corresponds to a section of the journey that is uphill
- Some pupils may think that the graph $y = x^2 + c$ is the graph of $y = x^2$ translated horizontally.

Understanding risk II — 8 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Probability progression map

- apply systematic listing strategies
- record describe and analyse the frequency of outcomes of probability experiments using frequency trees
- enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams
- construct theoretical possibility spaces for combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

Return to overview

Possible themes

- Explore experiments and outcomes
- Develop understanding of probability
- Use probability to make predictions

Possible key learning points

- List all elements in a combination of sets using a Venn diagram
- List outcomes of an event systematically
- Use a table to list all outcomes of an event
- Use frequency trees to record outcomes of probability experiments
- Construct theoretical possibility spaces for combined experiments with equally likely outcomes
- Calculate probabilities using a possibility space
- Use theoretical probability to calculate expected outcomes
- Use experimental probability to calculate expected outcomes

Prerequisites

- Convert between fractions, decimals and percentages
- Understand the use of the 0-1 scale to measure probability
- Work out theoretical probabilities for events with equally likely outcomes
- Know how to represent a probability
- Know that the sum of probabilities for all outcomes is 1

Mathematical language

Outcome
Event
Experiment, Combined experiment
Frequency tree
Enumerate
Set
Venn diagram
Possibility space, sample space
Equally likely outcomes
Theoretical probability
Random
Bias, Fairness
Relative frequency

Notation
P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

Pedagogical notes

The Venn diagram was invented by John Venn (1834 – 1923)
NCETM: Glossary

Common approaches
All students are taught to use 'DIME' probability recording charts
All classes carry out the 'race game' as a simulated horse race with horses numbered 1 to 12

Reasoning opportunities and probing questions

- Show me a way of listing all outcomes when two coins are flipped
- Convince me that there are more than 12 outcomes when two six-sided dice are rolled
- Convince me that 7 is the most likely total when two dice are rolled

Suggested activities

KM: Sample spaces
KM: Race game
Hwb: Q37, Q79
KM: Stick on the Maths L4HD3
NRICH: Prize Giving (frequency trees)

Possible misconceptions

- Some students may think that there are only three outcomes when two coins are flipped, or that there are only six outcomes when three coins are flipped
- Some students may think that there are 12 unique outcomes when two dice are rolled
- Some students may think that there are 12 possible totals when two dice are rolled

Presentation of data — *4 lessons*

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data
- use and interpret scatter graphs of bivariate data
- recognise correlation

Return to overview

Possible themes

- Explore types of data
- Construct and interpret graphs
- Select appropriate graphs and charts

Possible key learning points

- Construct and interpret a grouped frequency table for continuous data
- Construct and interpret histograms for grouped data with equal class intervals
- Plot a scatter diagram of bivariate data
- Interpret a scatter diagram using understanding of correlation

Prerequisites

- Know the meaning of discrete data
- Interpret and construct frequency tables
- Construct and interpret pictograms, bar charts, pie charts, tables and vertical line charts

Mathematical language

Data
Categorical data, Discrete data
Continuous data, Grouped data
Table, Frequency table
Frequency
Histogram
Scale, Graph
Axis, axes
Scatter graph (scatter diagram, scattergram, scatter plot)
Bivariate data
(Linear) Correlation
Positive correlation, Negative correlation

Notation
Correct use of inequality symbols when labeling groups in a frequency table

Pedagogical notes

The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is 'a diagram made of rectangles whose areas are proportional to the frequency of the group'. If the class widths are equal, as they are in this unit of work, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density.
Lines of best fit on scatter diagrams are not introduced until Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time.
NCETM: Glossary

Common approaches
All students collect data about their class's height and armspan when first constructing a scatter diagram

Reasoning opportunities and probing questions

- Show me a scatter graph with positive (negative, no) correlation. And another. And another.
- Kenny thinks that 'frequency diagram' is just a 'fancy' name for a bar chart. Do you agree with Kenny? Explain your answer.
- What's the same and what's different: scatter diagram, bar chart, pie chart?
- Always/Sometimes/Never: A scatter graph shows correlation

Suggested activities

KM: Make a 'human' scatter graph by asking pupils to stand at different points on a giant set of axes.
KM: Gathering data
KM: Spreadsheet statistics
KM: Stick on the Maths HD2: Selecting and constructing graphs and charts
KM: Stick on the Maths HD3: Working with grouped data

Possible misconceptions

- Some pupils may label the bar of a histogram rather than the boundaries of the bars
- Some pupils may think that there are gaps between the bars in a histogram
- Some pupils may misuse the inequality symbols when working with a grouped frequency table

Measuring data — 6 lessons

Key concepts (GCSE subject content statements)

The Big Picture: Statistics progression map

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)
- apply statistics to describe a population

Return to overview

Possible themes

- Investigate averages
- Explore ways of summarising data
- Analyse and compare sets of data

Possible key learning points

- Find the modal class of set of grouped data
- Find the class containing the median of a set of data
- Calculate an estimate of the mean from a grouped frequency table
- Estimate the range from a grouped frequency table
- Analyse and compare sets of data, appreciating the limitations of different statistics (mean, median, mode, range)
- Choose appropriate statistics to describe a set of data

Prerequisites

- Understand the mean, mode and median as measures of typicality (or location)
- Find the mean, median, mode and range of a set of data
- Find the mean, median, mode and range from a frequency table

Mathematical language

Average
Spread
Consistency
Mean
Median
Mode
Range
Statistic
Statistics
Approximate, Round
Calculate an estimate
Grouped frequency
Midpoint

Notation
Correct use of inequality symbols when labeling groups in a frequency table

Pedagogical notes

The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean').
NCETM: Glossary

Common approaches
Every classroom has a set of statistics posters on the wall
All students are taught to use mathematical presentation correctly when calculating and rounding solutions, e.g. (21 + 56 + 35 + 12) ÷ 30 = 124 ÷ 30 = 41.3 to 1 d.p.

Reasoning opportunities and probing questions

- Show me an example of an outlier. And another. And another.
- Convince me why the mean from a grouped set of data is only an estimate.
- What's the same and what's different: mean, modal class, median, range?
- Always/Sometimes/Never: A set of grouped data will have one modal class
- Convince me how to estimate the range for grouped data.

Suggested activities

KM: Swillions
KM: Lottery project
NRICH: Half a Minute

Possible misconceptions

- Some pupils may incorrectly estimate the mean by dividing the total by the numbers of groups rather than the total frequency.
- Some pupils may incorrectly think that there can only be one model class.
- Some pupils may incorrectly estimate the range of grouped data by subtracting the upper bound of the first group from the lower bound of the last group.